IDENTIFICATION OF BOUNDARY CONDITIONS FOR ELEMENTS OF THE FLOW-THROUGH PART OF A TURBINE IN A GAS-TURBINE ENGINE

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A method is proposed for identifying boundary conditions on the basis of analytical solution of the inverse heat conduction problem and use of the rules governing the displacement of isotherms.

It is known that the accuracy of calculation of the thermal stress state of the rotor blades of high-temperature gas turbines in gas-turbine engines depends to a large extent on the reliable specification of boundary conditions for the gas and cooling air. The wide range of cooling-system designs makes it necessary to experimentally establish criterional relations to describe convective heat transfer in the internal channels of blades. Here, we present methods of determining coefficients of convective heat transfer α on the basis of anaytical solution of a unidimensional inverse boundary-value problem of nonsteady heat conduction (ICP) and laws governing the displacement of isotherms for heat-sensing elements washed by a coolant.

1. The solution of the corresponding ICP in [1] was obtained to ensure experimental conditions whereby heat propagates in a flat heat-sensing element with a thermally-insulated rear part. The time change in temperature f(Fo) on the element was approximated by the polynomial

$$f(\mathrm{Fo}) = \sum_{k=0}^{l} a_k \mathrm{Fo}^k \tag{1}$$

The solution obtained in [1] gives the following values of temperature and heat flux on the coolant-washed surface:

$$T_w(\mathrm{Fo}) = \sum_{k=0}^{l} a_k \left[\sum_{j=1}^{k+1} \frac{k!\,\mathrm{Fo}^{k-j+1}}{(k-j+1)!\,(2j-2)!} \right], \tag{2}$$

$$q_w(\mathrm{Fo}) = -\lambda \frac{1}{\delta} \sum_{k=1}^{l} a_k \left[\sum_{j=1}^{k+1} \frac{k!\,\mathrm{Fo}^{k-j+1}}{(k-j+1)!\,(2j-3)!} \right]. \tag{3}$$

In the case of approximation of the function f(Fo) by an exponent:

$$f(\mathrm{Fo}) = A + B\exp(-c\mathrm{Fo}) \tag{4}$$

we also have

$$T_w(\mathrm{Fo}) = A + B\exp(-c\mathrm{Fo})\cos(\sqrt{c}), \tag{5}$$

$$q_w(\mathrm{Fo}) = -\lambda \frac{1}{\delta} B\exp(-c\mathrm{Fo})\sin(\sqrt{c})\sqrt{c}. \tag{6}$$

In model studies, the temperature of the coolant T_f(Fo) is known. Thus, the use of Eqs. (2) and (3) or (5) and (6) makes it possible to determine the function α(Fo) as

$$\alpha(\mathrm{Fo}) = q_w(\mathrm{Fo})/(T_w(\mathrm{Fo}) - T_f(\mathrm{Fo})). \tag{7}$$

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TABLE 1. Results of Testing of a Program to Determine α with a Known Coolant Temperature

Time Fo	Temp. of rear side of plate f (Fo), °C	Temp. of medium T_f, °C	Heat-transfer coeff. α, W/ (m^2 · K)
1,18	31,42	60	999,26
1,57	33,57	60	1009,72
1,96	35,56	60	1015,34
2,35	37,40	60	1014,71
2,75	39,11	60	1006,15

TABLE 2. Results of Testing of a Program to Determine α with an Unknown Coolant Temperature

Time Fo	Temp. of rear surface of plate f(Fo), °C	Bio criterion Bi	Heat-transfer coeff. α, W/(m^2 · K)
1,18	31,42	0,2092	976,09
1,57	33,57	0,2121	989,85
1,96	35,56	0,2143	1000,04
2,35	37,40	0,2155	1005,51
2,75	39,11	0,2153	1004,89

Measurement of T_f(Fo) on engine turbine blades undergoing cooling in a full-scale experiment is highly problematic. Moreover, in this case, it is impossible to use two heat-sensing elements to find α(Fo) without knowing T_f(Fo). This complication can be circumvented by the fact that, in solving the ICP, we can also identify the rate of displacement of the isotherms v(τ). This rate is determined as follows:

$$v(\tau) = \frac{dn}{d\tau} = -\left(\frac{\partial T}{\partial \tau}\right) \Big/ \left(\frac{\partial T}{\partial n}\right). \qquad (8)$$

In approximating f(Fo) in accordance with (1) and (4), using the results in [1], we have the corresponding values of v(Fo) on the washed surface of the heat-sensing element:

$$v_w(\text{Fo}) = \frac{a}{\delta} \frac{\sum_{k=1}^{l} a_k \left[\sum_{j=1}^{k} \frac{k!\,(\text{Fo})^{k-j}}{(k-j)!\,(2j-2)!} \right]}{\sum_{k=1}^{l} a_k \left[\sum_{j=2}^{k+1} \frac{k!\,\text{Fo}^{k-j+1}}{(k-j+1)!(2j-3)!} \right]}, \qquad (9)$$

$$v_w(\text{Fo}) = \frac{a}{\delta} \sqrt{c}\,\text{ctg}\,\sqrt{c}\,. \qquad (10)$$

Considering the established [2] dependence of v_w(Fo) on the time-constant Biot criterion at constant temperature T_f, having the following form for the stage during which the thermal kinetics are regularized throughout the body

$$v_w = \frac{\mu_1^2}{\text{Bi}} \frac{a}{\delta}, \qquad (11)$$

we can determine Bi (or α) from Eq. (11) if we calculate the left side of this equation in accordance with (9) or (10) and if the values of μ_1 in the right side of (11) are calculated as the first positive root of the characteristic equation

$$\mu = \text{arctg}(\text{Bi}/\mu). \qquad (12)$$

It should be noted that, after α is found, it is also possible to identify the temperature of the coolant T_f. Here, we make use of either Eqs. (2) and (3) or (5) and (6):

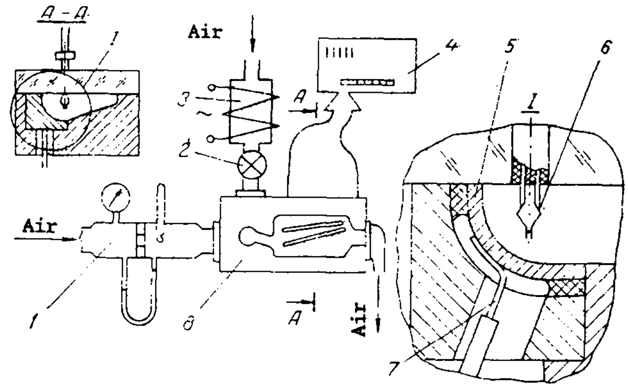

Fig. 1. Diagram of unit used to determine α on models of blades undergoing cooling: 1) section for measuring air velocity; 2) valve; 3) electric furnace; 4) oscillograph; 5) heat-sensing element; 6) thermocouple to measure ambient temperature; 7) thermocouple on the rear side of the plate; 8) model of blade.

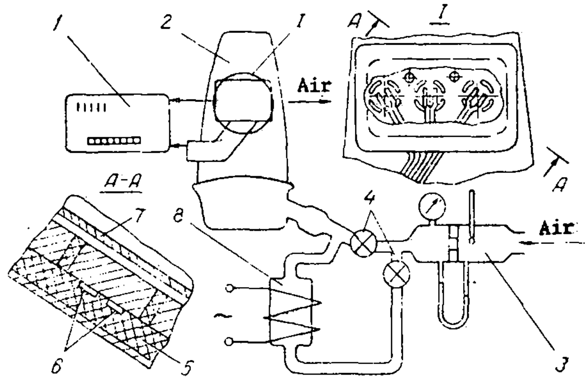

Fig. 2. Diagram of unit used to determine α on full-size blades undergoing cooling: 1) oscillograph; 2) turbine blade; 3) section for measurement of air velocity; 4) valve; 5) thermal insulation; 6) thermocouple to measure blade temperature; 7) blade deflector; 8) electric furnace.

$$T_f(\mathrm{Fo}) = T_w(\mathrm{Fo}) - q_w(\mathrm{Fo})/\alpha. \qquad (13)$$

Tables 1 and 2 show some of the results of numerical tests of programs developed to determine α on the basis of the above-described algorithms [we identified $\alpha = 1000$ W/(m²·K)].

Examination of the results of the numerical modeling shows that the proposed algorithms are highly effective and are suited for practical application.

2. Experiments to determine α for the coolant on the inlet edge of a model were conducted on a stand. A basic sketch of the stand is shown in Fig. 1. The model simulated half the internal cavity of a blade truncated along the middle line and rectified. The inlet edge was composed of a copper plate with a thickness $\delta = 0.5$ mm and had slits filled with epoxy resin to eliminate longitudinal heat flows. Chromel–Alumel thermocouples were soldered to the back side of the plate, which was then thermally insulated. Sensors to determine the temperature of the medium were placed downflow in the cover of the model.

A mixture of hot and cold air was passed through the model so that the plate was heated to 60-70°C. The flow of hot air was then cut off, and the cold air – whose flow rate corresponded to the test regime – cooled the plate.

Experiments to determine α in the cooling channels of full-scale rotor blades of turbines were conducted on the stand depicted in Fig. 2. Chromel–Alumel thermocouples were

TABLE 3. Values of α Established for the Face of the Blade without the Use of Values of T_f

Time Fo	Temp. of rear surface of blade f(Fo), °C	Biot criterion Bi	Heat-transfer coeff. α, $W/(m^2 \cdot K)$	Reynold criterion Re	Nusselt number Nu
0,55	47,46	0,2772	1178,00	4822	59,03
1,10	46,16	0,2771	1177,61	4832	59,15
1,65	44,86	0,2697	1146,19	4843	57,71
2,20	44,08	0,2520	1071,18	4853	54,06
...	...	...	...	...	...
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...	...	...	...	...	...
0,55	24,38	0,4163	1769,40	18026	94,81
1,10	23,89	0,4328	1839,36	18039	98,63
1,65	23,50	0,4341	1844,95	18057	99,04
2,20	23,17	0,4072	1730,50	18067	92,95

soldered to the outside surface of the blade (on the back edge and face). To prevent longitudinal flow of heat, we cut specially shaped slits and filled them with epoxy. The surface of the blade was then thermally insulated at the locations of the thermocouples.

In tests of the models and full-size blades, each coolant flow rate regime was repeated three times. This improved the accuracy of the functions f(Fo) after they were smoothed with the cubic spline or the exponent in accordance with (1) and (4). To improve the accuracy of the smoothing, we chose a certain part of the time range: we omitted the initial time section after a cutoff of the hot air of ~1 sec, judging this section to be unsteady in the gasdynamic sense; we also omitted the final time section, beginning at the moment when the difference in temperature between the sensor and the medium became comparable to the error of the recording instruments.

The results of tests of model and full-size blades, with the experimental data analyzed by the above-described methods, give stable values of α. This is shown in particular by the analysis in Table 3.

The test results were used to optimize blade-cooling regimes, while the criterional relations obtained to describe convective heat transfer on the coolant side were used to determine the thermal stress state of blades.

NOTATION

f(Fo) and T_w(Fo), T_f, temperature on the rear and washed sides of the heat-sensing element, coolant temperature; q_w, heat flux to the wall; α and $Bi = \alpha\delta/\lambda$, coefficient of convective heat transfer and Biot criterion; λ and a, thermal conductivity and diffusivity; τ and $Fo = a\tau/\delta^2$, time and Fourier number; δ, thickness of the heat-sensing element.

LITERATURE CITED

1. N. M. Tsirel'man, Inzh.-Fiz. Zh., 49, No. 6, 1015-1018 (1985).
2. N. M. Tsirel'man, Inzh.-Fiz. Zh., 25, No. 2, 349-352 (1973).